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Analysis of an N-policy M/M/1 queueing system with server start up, unreliable server and balking

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Abstract

This paper is concerned with the optimal control of a non reliable and removable server in an N-policy M/M/1 queueing system with server start up and balking. When the queue length reaches N (N≥1), the server is immediately turned on but is temporarily unavailable to serve the waiting units. The server needs some startup time before providing the service. It is assumed that server breakdowns according to a Poisson process and the repair time have an exponential distribution. Arriving customers may balk with a certain probability and may depart without getting service due to impatience. Lack of service occur where the server is busy or due to sudden breakdown. Explicit expression for the number of units in the system are obtained and also derived various system measures. The total expected cost function is developed to determine the optimal threshold N at a minimum cost. The sensitivity analysis is performed to examine the effect of various parameters in the system.

Keywords: N-policy, server breakdowns, M/M/1 queueing system, server startup, probability generating function, balking.

Introduction

Many practical queueing system especially those with balking and reneging have been widely applied to many real life problems such as the situations involving impatient telephone switch board customers, the hospital emerging room handling critical patients, the inventory system with storage of perishable goods, etc. Queueing system with balking, reneging or both have been studied by many researchers. Haight[1] first considered an M/M/1 queue with balking. An M/M/1 queue with customers reneging was also proposed by Haight[2]. The combined effects of balking and reneging in an M/M/1/N queue have been investigated by Ancker and Gafarian[3,4]. Barrer[5] carried out one of the early work on reneging where he considered deterministic reneging with single server Markovian arrival and service rates. Abou-El-Atc and Harri[6] considered multiple server queueing system M/M/C/N with balking and reneging. Wang and Chang[7] extended this work to study an M/M/R/N queueing system with balking, reneging and server breakdowns. Multi-server Markovian queuing system with balking and reneging was discussed by Choudhury and Medhi[8]. M/M/1 queueing system with the customers impatience in the form of random balking was analyzed by Manoharan and Jose[9]. This paper provides an adequate service for its customers with tolerable waiting, whenever customer impatience becomes sufficiently strong and customers leave before being served.

The concept of the *N* policy was first introduced by Yadin and Naor[10]. Wang[11] first proposed a management policy for Markovian queueing systems under the *N* policy with server breakdowns. Wang[12] and Wang *et al.*[13] extended the model proposed by Wang[11] to M/Ek/1 and M/H2/1 queueing systems, respectively. They developed the analytic closed form solutions and provided a sensitivity analysis. Jayachitra and James Albert[14] presented an elaborated survey on various queueing models under N-policy. Jayachitra and James Albert[15] discussed optimal control of an N-policy M/Ek/1 queueing system with server startup and breakdown. Also the authors[16,17] studied an Erlangian model under server breakdowns and multiple vacations and provided a cost model to determine the optimal operating policy at minimum cost. However to the best of our knowledge for M/M/1 queueing system with server startup, N-policy, unreliable server and balking, there is no literature which takes customers impatience into consideration. This motivates us to study a queueing system with N-policy, server startup, breakdowns and balking.

Thus in the present paper we consider an M/M/1 queueing system with server start up, N-policy, unreliable server and balking. The model can be realized in many real life situations.

The article is organized as follows. A full description of the model is given in section 2. The steady state analysis of the system state probabilities is performed through the generating functions in section 3. Steady state results on the expected number of customers in different states of the server are given in section 4. Special cases of this model are discussed in section 5. In section 6 the characteristic features of the system are investigated. Optimal control policy is explained in section 7. In section 8 sensitivity analysis is carried out through numerical illustration. Finally in section 9 conclusions and future scope of the study are presented.

The main objectives of the analysis carried out in this paper for the optimal control policy are: i. To establish the steady state equations and obtain the steady state probability distribution of the number of customers in the system in each state. ii. To derive expressions for the expected number of customers in the system when the server is in idle, startup, busy and breakdown states respectively. iii. To formulate the total expected cost functions for the system, and determine the optimal value of the control parameter N. iv. To carry out sensitivity analysis on the optimal value of N and the minimum expected cost for various system parameters through numerical experiment.

Model description

For the purpose of analytical investigation, we consider the model with the following assumptions: i. The arrival is poisson process with parameter λ. Arriving cusomers at the server form a single waiting line and is served in the order of their arrivals. The server may serve only one unit at a time and the service times are exponentially distributed with mean 1/μ. ii. The server is turned off when the system becomes empty. As soon as the number of customers in the system reaches N, the server is immediately turned on but is temporarily unavailable to the waiting units. It needs to take exponential startup time with parameter Θ. As soon as the startup period over, it serves the waiting units immediately. iii. When the server is working, it may breakdown at any time with a Poisson breakdown rate α. iv. When the server fails, it is immediately repaired at a repair rate β, where the repair times are exponentially distributed. v. A customer may balk from the queue with probability b_0 when the server is in idle or may balk with a probability b_1 when the server is in service made due to impatience.

Steady State Analysis

In steady state the following notations are used.

$P_{0,n,}$ = Probability that there are n customers in the system when the server is idle. There are two situations when the server is idle. i. the server is turned off if $n \le N-1$; ii. it is turned on and performing startup if $n \ge N$.

$P_{1,n}$ = Probability that there are n customers in the system when the server is turned on and in operation where n=1,2,3…

$P_{2,n}$ = Probability that there are n customers in the system when the server is in operation and found to be broken down where n=1,2,3…

The steady state equations are given as follows

$$\lambda b_0 P_{1,0} = \mu P_{1,1} \tag{1}$$

$$\lambda b_0 P_{0,n} = \lambda b_0 P_{0,n-1} \quad (1 \le n \le N-1) \tag{2}$$

$$(\lambda b_1 + \theta) P_{0,\mathrm{N}} = \lambda b_0 P_{0,\mathrm{N}-1} \tag{3}$$

$$(\lambda b_1 + \theta) P_{0,\mathrm{n}} = \lambda b_1 P_{0,n-1} \quad \mathrm{n} > \mathrm{N} \tag{4}$$

$$(\lambda b_1 + \mu + \alpha) P_{1,1} = \mu P_{1,2} + \beta P_{2,1} \tag{5}$$

$$(\lambda b_1 + \mu + \alpha) P_{1,n} = \lambda b_1 P_{1,n-1} + \mu P_{1,\mathrm{n}+1} + \beta P_{2,n} \ (2 \le n \le N-1) \tag{6}$$

$$(\lambda b_1 + \mu + \alpha) P_{1,n} = \lambda b_1 P_{1,n-1} + \mu P_{1,n+1} + \beta P_{2,n} + \theta P_{0,n} \ \ (n \ge N) \tag{7}$$

$$(\lambda b_1 + \beta) P_{2,1} = \alpha P_{1,1} \tag{8}$$

$$(\lambda b_1 + \beta) P_{2,n} = \lambda b_1 P_{2,n-1} + \alpha P_{1,n} \ \ (n \ge 2\ ) \tag{9}$$

Solving equations (1),(2),(3) and (4) recursively, we finally get

$$P_{0,n} = \begin{cases} P_{0,0}, & 1 \le n \le N-1 \\ R^{n-(N-1)} P_{0,0}, & n \ge N \end{cases} \tag{10}$$

Where $\mathrm{R} = \dfrac{\lambda b_0}{\lambda b_1 + \theta}$

Probability generating function: The technique of using the probability generating function may be applied in a recursive manner from equation (1) to (9) to obtain the analytic solution of $P_{0,0}$ in neat closed form expression. Define the probability generating function of $H_0(z)$, $H_1(z)$, Q(z)and R(z) respectively as follows:

$$H_0(z) = \sum_{n=0}^{N} z^n P_{0,\mathrm{n}} \tag{11}$$

$$H_1(z) = \sum_{n=N}^{\infty} z^n P_{0,n} \tag{12}$$

$$Q(z) = \sum_{i=1}^{\infty} z^n P_{1,n} \tag{13}$$

$$R(z) = \sum_{n=1}^{\infty} z^n P_{2,n} \tag{14}$$

Applying algebraic manipulation technique to equation (2) and using in equation (11), we get the following

$$H_0(z) = \left[\frac{1-z^N}{1-z} \right] P_{0,0} \tag{15}$$

Using equation (10) in equation (12) we get

$$H_1(z) = \left[\frac{Rz^N}{1-Rz} \right] P_{0,0} \tag{16}$$

Multiplying equation (3) by z and (4) by z^n and summary over n we get

$$H_1(z) = \left[\frac{\lambda b_0 z^N}{\lambda b_1(1-z)+\theta}\right] P_{0,0} \tag{17}$$

Equating (16) and (17) we get

$$\frac{Rz^N}{1-Rz} = \frac{\lambda b_0 z^N}{\lambda b_1(1-z)+\theta} \tag{17a}$$

Multiplying equation (1) by z and (5),(6)and (7) by z^n and summary over n we get

$$\left[\lambda b_1 z^2 - (\lambda b_1 + \mu + \alpha) z + \mu\right] Q(z) + \beta z R(z) = \left[\lambda b_0 z - \frac{\theta R z^{N+1}}{1-Rz}\right] P_{0,0} \tag{18}$$

Applying same procedure over the equations (8) and (9) to obtain the following equations

$$R(z) = \left[\frac{\alpha}{\lambda b_1 + \beta - \lambda b_1 z}\right] Q(z) \tag{19}$$

Substituting equation (19) in equation (18) we get

$$Q(z) = \left[\frac{\lambda b_1 + \beta - \lambda b_1 z}{\left[\lambda b_1 z^2 + (\lambda b_1 + \mu + \alpha) z + \mu\right]\left[\lambda b_1 + \beta - \lambda b_1 z\right] + \beta z \alpha}\right]\left[\lambda b_0 z - \frac{\theta R z^{N+1}}{1-Rz}\right] P_{0,0} \tag{20}$$

Probability that the empty system: We evaluate the probability $P_{0,0}$ using normalizing condition. For this purpose we evaluate $H_0(1), \mathrm{H}_1(1), \mathrm{Q}(1)$ and $R(1)$ from equations (15),(16),(20) and (19) respectively as

$$H_0(1) = NP_{0,0} \tag{21}$$

$$H_1(1) = \frac{\lambda b_0}{\theta} P_{0,0} \tag{22}$$

$$Q(1) = \left[\frac{\rho b_0\left(N + \frac{\lambda b_0}{\theta}\right)}{\left[1 - \rho b_1\left(1 + \frac{\alpha}{\beta}\right)\right]}\right] P_{0,0} \tag{23}$$

$$R(1) = \frac{\alpha}{\beta}\left[\frac{\rho b_0\left(N + \frac{\lambda b_0}{\theta}\right)}{\left[1 - \rho b_1\left(1 + \frac{\alpha}{\beta}\right)\right]}\right] P_{0,0} \tag{24}$$

Let G(Z) represent the number of customers in the system.

$$G(Z) = H_0(z) + H_1(z) + Q(z) + R(z) \tag{25}$$

Now using the normalizing condition $G(1) = 1$, we obtain the value of probability

$$P_{0,0} = \frac{1 - \rho b_1\left(1 + \frac{\alpha}{\beta}\right)}{\left(N + \frac{\lambda b_0}{\theta}\right)\left(1 - \rho\left(1 + \frac{\alpha}{\beta}\right)(b_1 - b_0)\right)} \quad \text{where } \rho = \frac{\lambda}{\mu} \tag{26}$$

Let P_0, P_1, P_2 and P_3 be the probability that the server is in idle, startup, busy and break down states respectively. Then,

$P_0 = H_0(1)$ (i)

$P_1 = H_1(1)$ (ii)

$P_2 = Q(1)$ (iii)

$P_3 = R(1)$ (iv)

Expected number of customers in the system

We define the expected number of customers in the system as follows.

L_I = the expected number of customers in the system when the server is idle.

L_s = the expected number of customers in the system when the server is doing pre service (start up work).

L_w = the expected number of customers in the system when the server is working.

L_d = the expected number of customers in the system when the server is broken down.

L_N = Expected number of customer in the system.

Using the probability generating functions expected number of units in the system at different states are presented below.

$$L_I = H_0'(1) = \frac{N(N-1)}{2} P_{0,0} \tag{27}$$

$$L_s = H_1'(1) = \frac{\lambda b_0}{\theta}\left[N + \frac{\lambda b_0}{\theta}\right] P_{0,0} \tag{28}$$

$$L_w = Q'(1)$$

$$L_w = \left\{\frac{\rho b_0\left(\left[2\frac{\lambda b_0}{\theta} + N(N+1) - \frac{2\lambda b_1}{\beta}\left(N + \frac{\lambda b_0}{\theta}\right)\right]\right)}{2\left[1 - \rho b_1\left(1 + \frac{\alpha}{\beta}\right)\right]} + \frac{\rho b_0 \lambda b_0\left(N + \frac{\lambda b_0}{\theta}\right)}{\theta\left[1 - \rho b_1\left(1 + \frac{\alpha}{\beta}\right)\right]} - \frac{\rho b_0\left(N + \frac{\lambda b_0}{\theta}\right)\lambda b_1\left[\lambda b_1 - \mu - \alpha - \beta\right]}{\beta\left[1 - \rho b_1\left(1 + \frac{\alpha}{\beta}\right)\right]^2}\right\} P_{0,0} \tag{29}$$

$$L_d = R_0'(1) = \frac{\alpha}{\beta} L_w + Q(1)\left(\frac{-\alpha \lambda b_1}{\beta}\right) \tag{30}$$

The expected number of customers in the system is given by

$$L_N = H_0'(1) + H_1'(1) + Q'(1) + \mathrm{R}'(z)$$

$$L_N = \left[\begin{array}{l} \dfrac{N(N-1)}{2} + \dfrac{\lambda b_0}{\theta}\left(N + \dfrac{\lambda b_0}{\theta}\right) + \left\{ \dfrac{\rho b_0 \left[2\dfrac{\lambda b_0}{\theta} + N(N+1) - \dfrac{2\lambda b_1}{\beta}\left(N + \dfrac{\lambda b_0}{\theta}\right)\right]}{2\left[1 - \rho b_1\left(1 + \dfrac{\alpha}{\beta}\right)\right]} + \dfrac{\rho b_0 \lambda b_0 \left(N + \dfrac{\lambda b_0}{\theta}\right)}{\theta\left[1 - \rho b_1\left(1 + \dfrac{\alpha}{\beta}\right)\right]} - \dfrac{\rho b_0 \left(N + \dfrac{\lambda b_0}{\theta}\right)\lambda b_1 \left[\lambda b_1 - \mu - \alpha - \beta\right]}{\beta\left[1 - \rho b_1\left(1 + \dfrac{\alpha}{\beta}\right)\right]^2} \right\}\left(1 + \dfrac{\alpha}{\beta}\right) + \\ \dfrac{\rho b_0}{\left[1 - \rho\left(1 + \dfrac{\alpha}{\beta}\right)(b_1 - b_0)\right]}\left(\dfrac{-\alpha\lambda b_1}{\beta}\right) \end{array} \right] P_{0,0} \tag{31}$$

Special Cases

In this section we present some existing results in the literature which are special cases of our model.

Case (i): If $\Theta=\infty, \alpha=0, \beta=\infty$, $b_1=1$, $b_0=1$ and $N=1$, our model represents the ordinary M/M/1 queueing system with reliable case. When N=1, $\Theta=\infty$, $\alpha=0$, $b_1=1$, $b_0=1$ and $\beta=\infty$, expression (20) for Q(z) reduces to the special case stated in the expressions (2.14) of Gross and Harris[18].

Case (ii): If $\Theta=\infty$, $b_1=1$ and $b_0=1$ our model represents the N-policy M/M/1 queueing system with unreliable case. When $\Theta=\infty$, $b_1=1$ and $b_0=1$ expression (20) for Q(z) and (19) for R(z) reduces to a special case of expression (18) and (19) of Wang[11] respectively.

Case (iii): If $\Theta=\infty$, $\alpha=0$, $\beta=\infty$, $b_1=1$, $b_0=1$ and $N=1$,equation (26) reduces to the result of probability that the empty system in the ordinary M/M/1 queueing model.

Other System Characteristic

Let I, S, B, and D denote the length of the server off, startup, busy and breakdown periods respectively. Applying the memory less property of the Poisson process, we find that the mean length of the idle period is

$$E(I) = \frac{N}{\lambda b_0} \tag{32}$$

The expected length of the idle period, startup period, busy period and breakdown period are denoted by E(I), E(S),E(B) and E(D) respectively. The expected length of the busy cycle is given by

E(C) = E(I) + E(S)+E(B)+E(D)

From equations (21) to (24) we obtain the long run fraction of time for the server is idle, start up, busy and broken down respectively.

$$\frac{E(I)}{E(C)} = H_0(1) = NP_{0,0} \tag{33}$$

$$\frac{E(\mathrm{S})}{E(C)} = H_s(1) = \frac{\lambda b_0}{\theta} P_{0,0} \tag{34}$$

$$\frac{E(B)}{E(C)} = Q(1) = \frac{\rho b_0}{1 - \left[\rho\left(1 + \dfrac{\alpha}{\beta}\right)(\mathrm{b}_1 - b_0)\right]} \tag{35}$$

$$\frac{E(D)}{E(C)} = R(1) = \frac{\alpha}{\beta} Q(1) \tag{36}$$

From equation, (32) and (33) we have the number of busy cycles per unit time

$$\frac{1}{E(C)} = \lambda b_0 P_{0,0} \tag{37}$$

Optimal Control policy

We develop the total expected cost function per unit time for the N-policy $M/\mathrm{M}/1$ Queueing system with server startup, unreliable server and balking in which N is a decision variable. With the cost structure being constructed, the objective is to determine the optimal operating policy so as to minimize this function. Let

C_h = holding cost per unit time for each customer present in the system.

C_o = Cost per unit time for keeping the server idle.

C_f =Set up cost per busy cycle.

C_d =breakdown cost per unit time

C_s = startup cost per unit time for preparatory work of the server before starting the service.

C_b =cost per unit time for the server keeping on and in operation.

C_k = Cost per unit time when a customer balks.

$$T_{\cos t}(N) = C_h L_N + \frac{C_f}{E(C)} + C_o \frac{E(I)}{E(C)} + C_s \frac{E(S)}{E(C)} + C_b \frac{E(B)}{E(C)} + C_d \frac{E(D)}{E(C)} + C_k \left\{ \lambda(1-b_0)\frac{E(I)}{E(C)} + \lambda(1-b_1)\left[\frac{E(S)}{E(C)} + \frac{E(B)}{E(C)} + \frac{E(D)}{E(C)}\right]\right\} \quad (38)$$

Using equations (31), (33)-(37), the results of (38) can be explicitly expressed which is a very long and complex formula for $T_{\cos t}(N)$. We obtain the optimal value N which minimizes the cost function $T_{\cos t}(N)$, by differentiating it with respect to N and setting the result to be zero.

i.e, $$\frac{\partial}{\partial N} T_{\cos t}(N) = 0 \quad (39)$$

The solution N to (39) may not be an integer and the optimal positive integer value of N is one of the integers surrounding N* which gives a smaller cost $T_{\cos t}$. Here it should be pointed out explicitly that the solution really gives the minimum value and $\frac{\partial^2}{\partial N^2} T_{\cos t}(N) = 0$ at N=N* is greater than zero when the values of system parameters satisfy suitable conditions. However it is quite tedious to present the explicit expression. Therefore we will perform the numerical illustrations to demonstrate that the function is really convex and the solution gives a minimum.

Sensitivity Analysis

In the course of analysis, sensitivity analysis has been carried out to find the optimum value of N (ie., N*) and the minimum cost based on changes in the system parameters using MATLAB. In order to arrive at the conclusions, the following arbitrary values of the system parameters are considered.

Case 1: C_h=5, C_0=10, C_b=100, C_d=200, C_s=125, C_f=500, C_k=10

Case 2: C_h=5, C_0=20, C_b=200, C_d=250, C_s=250, C_f=500, C_k=15

Case 3: C_h=5, C_0=50, C_b=400, C_d=350, C_s=500, C_f=1000, C_k=40

Case 4: C_h=10, C_0=100, C_b=500, C_d=400, C_s=600, C_f=1000, C_k=70

Case 5: C_h=50, C_0=100, C_b=500, C_d=500, C_s=800, C_f=1500, C_k=100

Table-1: The optimal value of N and its minimum expected cost for λ= 0.3, μ =1.2, β =3.0, b_0=0.4, b_1=0.2

	(Θ,α)	(0.4,0.2)	0.6,0.2)	(0.8,0.2)	(0.5,0.1)	(0.5,0.2)	(0.5,0.3)
Case(i)	N*	6	5	5	5	5	5
	T_{cost}(N*)	50.997	49.783	49.663	48.890	49.491	50.091
Case(ii)	N*	6	6	5	4	4	4
	T_{cost}(N*)	65.113	64.946	63.032	63.615	64.765	65.913
Case(iii)	N*	9	9	8	9	9	9
	T_{cost}(N*)	72.227	72.090	70.211	70.306	71.447	72.585
Case(iv)	N*	7	6	6	7	7	7
	T_{cost}(N*)	82.343	80.461	79.887	78.758	79.892	81.025
Case(v)	N*	4	3	3	3	3	3
	T_{cost}(N*)	94.192	90.872	90.662	90.780	91.895	93.009

Table-2: The optimal value of N and its minimum expected cost for $\lambda = 0.3$, $\Theta = 0.2$, $\alpha = 0.05$, $b_0 = 0.4$, $b_1 = 0.2$

	(μ,β)	(1.0,3.0)	1.2,3.0)	(1.4,3.0)	(1.4,2.0)	(1.4,4.0)	(1.4,6.0)
Case(i)	N*	7	8	9	8	8	8
	$T_{cost}(N^*)$	55.168	53.874	52.946	53.956	53.846	53.843
Case(ii)	N*	8	9	10	8	8	8
	$T_{cost}(N^*)$	71.432	69.442	67.972	69.649	69.321	69.249
Case(iii)	N*	10	11	12	11	11	11
	$T_{cost}(N^*)$	80.470	77.616	75.547	75.786	75.428	75.309
Case(iv)	N*	9	10	11	9	9	9
	$T_{cost}(N^*)$	86.724	83.940	81.927	82.163	81.809	80.611
Case(v)	N*	5	7	10	7	7	7
	$T_{cost}(N^*)$	111.592	109.705	107.853	108.060	107.750	107.644

Table-3: The optimal value of N and its minimum expected cost for $\beta = 3$, $\mu = 1.2$, $\alpha = 0.05$, $b_0 = 0.4$, $b_1 = 0.2$.

	(λ,Θ)	(0.2,04)	(0.4,0.4)	(0.6,0.4)	(0.4,0.1)	(0.4,0.3)	(0.4,0.5)
Case(i)	N*	8	10	11	10	10	8
	$T_{cost}(N^*)$	53.874	63.232	71.740	82.362	63.232	57.924
Case(ii)	N*	8	10	12	11	11	7
	$T_{cost}(N^*)$	69.422	80.445	90.525	101.934	80.449	74.375
Case(iii)	N*	10	11	15	13	13	10
	$T_{cost}(N^*)$	77.766	90.811	102.790	115.349	90.811	83.666
Case(iv)	N*	9	11	12	11	11	10
	$T_{cost}(N^*)$	84.059	97.910	110.699	121.593	97.980	91.623
Case(v)	N*	7	9	10	9	9	8
	$T_{cost}(N^*)$	109.703	130.855	149.594	181.397	130.855	115.232

Table-4: The optimal value of N and its minimum expected cost for $\lambda = 0.3$, $\mu = 1.2$, $\beta = 3.0$, $\alpha = 0.05$, $\Theta = 0.4$

	(b_0,b_1)	(0.4,0.3)	(0.45,0.3)	(0.5,0.3)	(0.4,0.2)	(0.4,0.3)	(0.4,0.4)
Case(i)	N*	5	5	6	5	5	5
	$T_{cost}(N^*)$	48.928	51.203	53.450	49.779	48.928	47.092
Case(ii)	N*	5	6	6	5	5	4
	$T_{cost}(N^*)$	73.580	77.015	80.399	74.236	73.580	72.949
Case(iii)	N*	8	9	10	8	8	7
	$T_{cost}(N^*)$	138.820	144.435	149.959	139.197	134.820	130.484
Case(iv)	N*	6	6	7	7	7	7
	$T_{cost}(N^*)$	220.275	228.966	237.548	229.445	220.275	211.158
Case(v)	N*	4	4	3	4	3	3
	$T_{cost}(N^*)$	427.812	461.951	495.795	435.099	427.812	420.686

The optimal value of N and its minimum expected cost $T_{coat}(N^*)$ for the above five cases are shown in Table-1 for various values of (Θ,α). It is observed that: i. T_{cost} (N*) decreases as Θ increases and $T_{cost}(N^*)$ increases as α increases. ii. N* decreases as Θ increases. Intuitively N* is insensitive to the changes in α. The optimal value of N and its minimum expected cost $T_{cost}(N^*)$ for the above five cases are shown in Table-2 for various values of (μ,β). From Table-2 we find that i. N* increases when μ changes from 1.0 to1.4. and N* remains unchanged when β changes from 2.0 to 6.0. ii) $T_{cost}(N^*)$ decreases as μ and β increases respectively. The optimal value of N and its minimum expected cost $T_{cost}(N^*)$ for the above five cases are shown in table (3)for various values of (λ ,r). From Table-3 we find that i. N* increases when λ increases and N*decreases when Θ increases. ii. $T_{cost}(N^*)$ increases when λ increases and $T_{cost}(N^*)$ decreases as Θ increases. From Table-4 we find that i. N* slightly increases when b_0 increases from 0.4 to 0.5 and N* slightly decreases when b_1 changes from 0.2 to 0.4. ii. $T_{cost}(N^*)$ increases when b_0 increases and $T_{cost}(N^*)$ decreases when b_1 increases.

Over all we conclude that: i. α, and β do not affect N*, ii. Θ, b_0 and b_1 rarely affect N*. iii. λ and μ affect N* significantly, iv. C_h and C_f have much stronger effect on N*

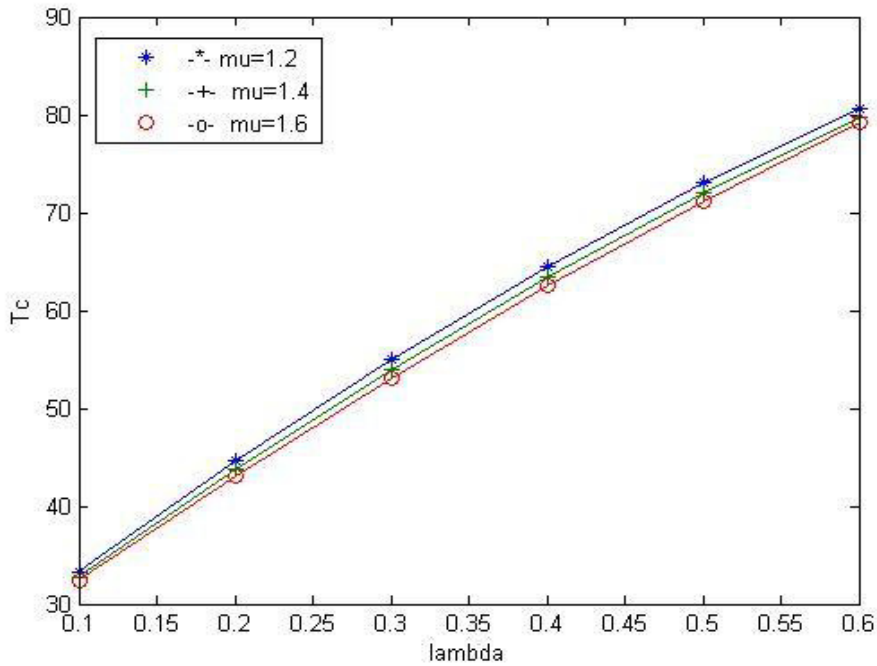

Figure-1: Arrival rate λ Vs Total cost Tc by varying service rate μ.

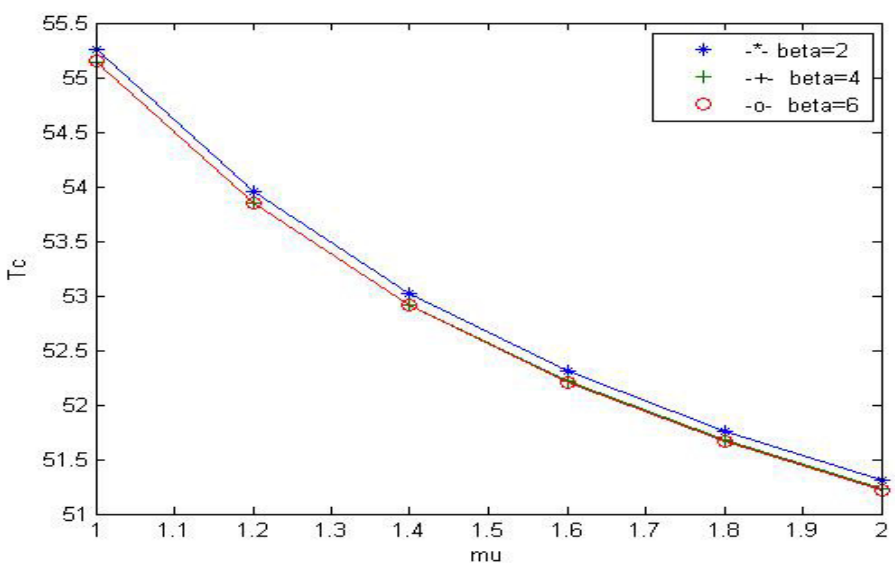

Figure-2: Service rate μ Vs Total cost T_c by varying repair rate β.

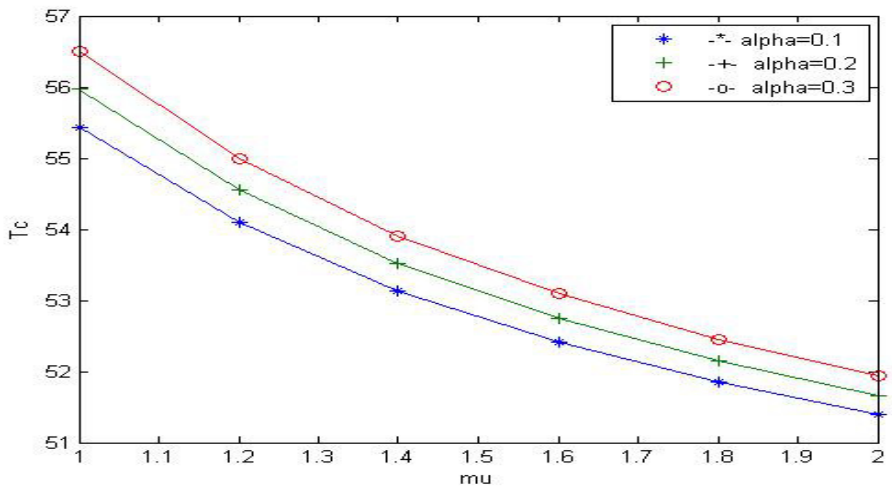

Figure-3: Service rate Vs Total cost by varying α.

Figure-1 displays the correlation between the arrival rate λ and the total cost (T_c) by varying service rate μ. We can observe that as the service rate goes on increasing, the total cost goes on decreasing. In Figure-2 the total cost goes on decreasing as we go on increasing the repair rate β. In Figure-3 the total cost goes on increasing as we go on increasing the breakdown rate α.

Conclusion

Optimal strategy analysis of an N-policy M/M/1 queueing system with server startup, unreliable server and balking is studied. Some of the system performance measures have been derived. A cost function has been formulated to determine the optimal value of N. Sensitivity analysis has been carried out through numerical illustrations. These numerical values will be useful in analyzing practical queueing system and make decision The present study can be extended by considering that the service time, breakdown time and repair time follow general distribution.

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